ABSTRACT

A towed array unavoidably detects its own tow submarine. This can in principle be exploited to detect and quantify unexpected increases in noise radiating from the submarine that increase its vulnerability to detection by threat sonar. Other potential applications are to image and localise new sound sources on the submarine and at the same time gain more accurate information on the shape of the towed array so that target detection remains possible throughout a manoeuvre. This paper presents techniques for near field acoustic imaging of the submarine and simultaneous improvement in the towed array shape determination and localisation. Simulated submarine sound sources and a U-turn are used to demonstrate these techniques.

1 INTRODUCTION

A towed array is an essential sensor for submarine operations to detect and classify targets at long ranges. However, they have the limitation of uncertain hydrophone positions making exploitation of higher gain adaptive beamforming techniques less effective. A towed array’s acoustic section becomes less linear during a turn which generally degrades the array gain and may prevent target tracking through the manoeuvre and for some time afterwards until the array resumes a stable shape. A magnetic heading sensor and a sinusoidal approximation of the towed array shape were shown to improve adaptive beamforming performance if the array shape distortion is sufficiently small and slowly changing [1]. Gerstoft et al [2] have shown that a surface tow vessel’s GPS track and the water pulley approximation for the towed array is accurate enough to use a white noise constrained adaptive beamformer.

Bucker [3] originally showed that conventionally beamformed acoustic signals from a distant target can be used to estimate the towed array shape by maximising the “sharpness” which is the sum of the squares of the beam power over all directions. Ferguson [4] demonstrated that the “sharpness” technique also works for adaptive beamforming although much more sensitive than conventional beamforming to deviations of the array from linear.

A requirement of the “sharpness” method is at least one far field target giving plane waves of sufficiently high SNR at the array. This is a disadvantage in a search for a quiet threat submarine with no other vessels present to provide reference plane waves. Our alternative idea is to exploit noise from the tow submarine in near field to extract the towed array shape and location. Focussing the array onto the towing submarine and adjusting the array shape and position to maximise a measure of the acoustic image contrast would also increase the SNR for far field targets.

It is also an attractive possibility that near field imaging might allow dispensing with heading and depth sensors on the towed array since its position relative to the submarine may be already determined accurately enough acoustically. Heading and depth sensors are needed to get the acoustic section orientation relative to the submarine and simple hydrodynamic approximations like water pulley do not account for wake and current shear affecting cant and kite angles. Further, near field acoustic imaging would allow continuous monitoring of the submarine source level so that any unexpected change to its signature can be quickly detected and its vulnerability to detection reassessed.

Much of the ground work for this paper was laid in an investigation of vessel self-ranging with its own towed array [5, 6]. Acoustic beacons operating between 3.8 and 19.2 kHz on a tug boat tracked every hydrophone of a Narama towed array through a U-turn[5]. High frequency beacons were possible owing to the array being analogue rather than digital with typical low sampling rates. Battle et al [7] and Duncan [5] have shown how matched field processing of a vessel’s multipath transmitted noise provides information on a towed array’s vertical shape profile.
Low frequencies from a submarine’s own noise rather than high frequency beacons will need to be used in any realistic acoustic system for towed array shapes. Noise sources radiated from known sources on a submarine, such as the propeller, is detectable in near field but the location along the hull from which other sources radiate most strongly may not be known very accurately. For this study we compare the near field image contrast maximisation results for both unknown and known acoustic source positions.

2 TOWED ARRAY PARAMETERISATION

For simplicity this investigation is confined to 2-D with the submarine and towed array in the same plane. The shape of the acoustic section of the array is defined in terms of a local coordinate system with its x axis connecting the end points of the predicted shape and its y axis intersecting the mid-point. As Figure 1 shows, the array shape parameters are the x and y offsets of one end of the array, normalised by dividing by the length L, $\delta_x / L$ and $\delta_y / L$, and the coefficients of a polynomial representing the difference in tangent direction of the two array shapes as a function of distance $s$ along the array, i.e.

$$\delta \theta(s) = \theta'(s) - \theta(s) = c_0 + c_1 \left( \frac{s}{L} \right) + c_2 \left( \frac{s}{L} \right)^2 + \cdots (1)$$

This method was trialled using a 2-D hydrodynamic simulation of a towed array [5] in which the tow-submarine at speed 2 m/s carried out a U-turn manoeuvre. The simulation was run using a normal drag coefficient $C_{DW}$ of 1.1 and a turn radius of 50 m to generate the assumed or predicted array shape, and then with a drag coefficient of 0.7 and a turn radius of 55 m to generate a ‘true’ array shape. Table 1 shows the model towed array parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (cm)</td>
<td>4</td>
</tr>
<tr>
<td>Tow cable (m)</td>
<td>93</td>
</tr>
<tr>
<td>forward VIM (m)</td>
<td>130</td>
</tr>
<tr>
<td>aft VIM (m)</td>
<td>50</td>
</tr>
<tr>
<td>Acoustic aperture (m)</td>
<td>110</td>
</tr>
<tr>
<td>Number of hydrophones</td>
<td>24</td>
</tr>
</tbody>
</table>

Figure 2 shows the difference of true and assumed towed array positions relative to the submarine at several times after starting the turn. Tow cable (magenta), vibration isolation modules (VIM) (blue) and acoustic aperture (red) are shown. Blue arrows point from assumed positions to true positions on the array.

A least squares fit to the differences between these two array shapes was then used to determine the time-dependent polynomial coefficients $c_0, c_1, c_2$ … which were in turn used to calculate a fitted array shape. The residuals plotted in Figure 3 as a function of time show how the sizes of the RMS position errors between the fitted and true array shapes due to the polynomial approximation vary through the manoeuvre, and have a sharp peak at around 230 seconds.

Experiments with different polynomial orders showed that the array shape is adequately parameterised by a 3rd order polynomial requiring just 6 parameters $c_0, c_1, c_2, c_3, \delta_x / L$ and $\delta_y / L$ to describe the deviation of the derived array shape, resulting from the acoustic analysis, from a predicted shape.

Our concept evaluation method is to use the correct towed array shape to simulate the hydrophone outputs due to signals from the tow submarine, and use the hydrodynamic model with incorrect drag coefficient and turn radius for the predicted towed array shape. Our aim is to detect and correct the predicted shape and location from the acoustic data alone and show that the corrected results are closer to the true shape.
3 AUTOFOCUSSING - UNKNOWN SOURCE LOCATIONS

For our measures of the focussed acoustic image contrast we chose concepts already developed in image processing. Autofocus algorithms compensate for small changes in synthetic aperture sonar (SAS) tracks that would otherwise significantly degrade the sonar image. Rafik [8] describes a number of SAS autofocus algorithms. Fortune et al [9] introduces a SAS autofocus technique based on contrast maximisation. She et al [10] uses a beamforming analogy for autofocus inverse SAR images.

After some experimentation we chose to use a modified version of the second order contrast (SOC) given in Fortune et al [9]. This measure at frequency \( f_j \) is

\[
C_2(f_j) = \frac{M \sum_{m=1}^{M} P_m^2(f_j)}{\left( \sum_{m=1}^{M} P_m(f_j) \right)^2}
\]

where \( M \) is the number of focal points, and

\[
P_m(f_j) = ss^* w_m^H a_0 a_0^H w_m
\]

is the expected output power for a beamformer with weight vector \( w_m \), focussed on point \( m \), due to a point source with amplitude \( s \) at the location defined by steering vector \( a_0 \). The source power is estimated from the received signal vector, \( x \), using \( ss^* \approx \frac{x^H x}{a_0^H a_0} \).

\( C_2(f_j) \) varies from 1, when all focal points have the same power, to \( M \) when all focal points except one have zero power. \( C_2 \) is similar to the “sharpness” which has a sum over look directions rather than a sum over focal points.

Combining contrasts at \( J \) different frequencies can be done using a weighted average

\[
\overline{C}_2 = \frac{\sum_{j=1}^{J} \alpha_j C_2(f_j)}{\sum_{j=1}^{J} \alpha_j}
\]

We chose weights \( \alpha_j \) to be a sum over focal point powers with the possibility of greater contributions from higher frequencies because they have better spatial resolution. Thus

\[
\alpha_j = f_j^\beta \sum_{m=1}^{M} P_m(f_j)
\]

where \( \beta \) is a constant.

Maximising the contrast \( \overline{C}_2 \) needs to correspond to minimising a cost function \( E \) in the optimisation process. A simple choice is
\[ E_C = \frac{1}{C_2} \] (6)

Other forms of cost functions derived from \( C_2 \) were investigated but produce similar results to (6) and are not reported on here.

Equations (2) – (6) apply to one “snapshot” of the focussed beam spectrum. Better accuracy for the towed array shape and location would be expected at the end of a sequence of snapshots. This is implemented by averaging \( \bar{C}_2 \) over the snapshots and using this averaged contrast in (6).

4 AUTOFOCUSING - KNOWN SOURCE LOCATIONS

One would expect the towed array shape is more accurately determined if some acoustic sources have known positions along the submarine’s hull. One may for instance exploit the propeller which is usually a significant noise source. Known locations enable use of the deterministic maximum likelihood (DML) method as the cost function. DML was described by Krim and Viberg [11] for angle of arrival estimation and here it is generalised to allow for generic array shape parameters and relative motion between the source and the hydrophone array.

The signal model is

\[ \mathbf{x}_k = A_k(\theta) \mathbf{s}_k + \mathbf{n}_k \] (7)

where \( \mathbf{x}_k \) is the array output vector for \( N \) hydrophones at the frequency of interest for snapshot \( k \), \( A_k(\theta) \) is the array steering matrix for snapshot, \( k \) corresponding to the known locations of the point sources, assuming array parameter vector \( \theta \), \( \mathbf{s}_k \) is a vector of unknown deterministic source complex amplitudes, and \( \mathbf{n}_k \) is a sensor noise vector assumed to be Gaussian, spatially uncorrelated and circularly symmetric with variance \( \sigma^2 \) for each hydrophone. \( \theta \) is a constant vector describing the array shape in the centre of the data observation period.

The log likelihood function for \( K \) snapshots

\[ l(\theta, \mathbf{s}_k, \sigma^2) = N \sigma^2 + \frac{1}{\sigma^2} \sum_{k=1}^{K} \left| \mathbf{x}_k - A_k(\theta) \mathbf{s}_k \right|^2 \] (8)

is minimised by differentiating by the signal vectors leading to

\[ \mathbf{s}_k = A_k^H(\theta) \mathbf{x}_k \] (9)

where

\[ A_k^H(\theta) \left( A_k(\theta) A_k(\theta) \right) = A_k^H(\theta) \] (10)

Substituting (9) into (8), and normalising by the array output power \( \mathbf{x}_k^H \mathbf{x}_k \), leads to the cost function

\[ E_{DML} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}_k^H \Pi_k(\theta) \mathbf{x}_k \] (11)

where

\[ \Pi_k(\theta) = I - A_k(\theta) A_k^H(\theta) \] (12)

and \( I \) is the identity matrix.

Sources at different frequencies can be utilised by forming a weighted average of their DML costs.

5 COST FUNCTION REGULARISATION AND MINIMISATION

Minimising a cost function may produce similar, and hence ambiguous, solutions. This problem can be reduced with a modified cost function that is biased to favour solutions closest to the predicted array shape. A convenient way of achieving this is to penalise nonzero array shape correction parameters. For the parameter vector:

\[ \theta = \left[ \begin{array}{c} \delta_0 \delta_0 \delta_0 \cdots \end{array} \right]^T \]

this involves using the combined cost function:

\[ E_T = E + \frac{\gamma}{N} \theta^T \mathbf{W} \theta \] (13)

where \( \gamma \) is a cost trade-off parameter and \( \mathbf{W} \) is a \( N \times N \) diagonal matrix determining the relative weights of \( N \) different array shape parameters.

Other possible regularisation techniques were considered but not used since Equation (13) worked well for the towed array application. Further, it was found that the reduced gradient optimisation method [12] was adequate for finding the cost function minimum, hence avoiding the greater computational complexity of techniques for overcoming ambiguous minima such as simulated annealing and genetic optimisation.

6 FOCUSED BEAMFORMING

Previous work on self ranging a vessel with its own towed array [5, 6] compared the acoustic image resolution and source level measurement by different focussed beamforming algorithms. The least squares minimum constrained (LSMC) beamformer gave overall superior results to minimum variance distortion-
less response (MVDR), spatially averaged MVDR and regularised inversion (RI) for imaging the tow vessel in a circle path scenario[6]. Hence this study limited itself to the LSMC.

Dropping the snapshot subscript, the beamformer output is

\[ y = w^H x \]  

(14)

Here \( w \) is the weight vector for the LSMC which is related to a focal point steering vector \( a_0 \) and is given by

\[ w = \frac{M^{-1}a_0}{a_0^HM^{-1}a_0} \]  

(15)

where

\[ M = AA^H + \varepsilon I \]  

(16)

As with Equation (7), \( A \) in (16) is the steering matrix to the known source positions. The white noise parameter \( \varepsilon \) is much less than 1 and stabilises the inversion of matrix \( M \).

7 SENSITIVITY OF BEAMFORMER OUTPUTS TO ARRAY SHAPE

Four acoustic sources were simulated at different positions, frequencies and source levels as in Table 2.

<table>
<thead>
<tr>
<th>Source</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental (Hz)</td>
<td>10</td>
<td>20</td>
<td>57</td>
<td>79</td>
</tr>
<tr>
<td>No. harmonics</td>
<td>100</td>
<td>50</td>
<td>17</td>
<td>12</td>
</tr>
<tr>
<td>RMS amplitude</td>
<td>100</td>
<td>70</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>X coordinate (m)</td>
<td>15</td>
<td>22</td>
<td>2</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 2. Simulated acoustic sources. Only subsets of the harmonics were used to calculate cost functions.

The origin of the local coordinate system (X, Y, Z) is at the stern tow point with the X direction along the submarine heading. All sources are on the centre line of the submarine (i.e. Y=0, Z=0). The source amplitudes are defined at 1 metre distance and apply to each individual harmonic. Spherical spreading along the direct path from source to hydrophone is used and multipath is omitted.

Note that the signals include frequencies well above the design frequency (150 Hz) of the acoustic section (Table 1). This is achievable because focusing towards the submarine, and the array curvature, usually keep aliases out of the small scope of directions being scanned by the array.

Figure 4 compares LSMC outputs incoherently averaged over interval 135s to 145s of the above towed array U-turn scenario using assumed and true hydrophone locations. The two strongest sources in Table 2 are detected and their locations are marked by green and magenta vertical lines. The assumed array location and shape result in poorer spatial resolution and incorrect positions of these two sources. Below we test if this is also reflected in higher values of the cost function.

8 COST FUNCTION PERFORMANCES

Figure 5 compares the two cost functions from Sections 3 and 4 for the time period 135s to 145s. SOC results were calculated by averaging costs for frequencies of 100Hz, 200Hz, 500Hz, and 980Hz using a weighting proportional to the square of the frequency and the beam power, i.e. Equation (5), with \( \beta = 2 \). DML used an unweighted average of costs calculated at different frequencies.

SOC and DML have similar sensitivity to Y direction (sideways) array displacements, but unsurprisingly DML is much more sensitive to X direction (lengthways) array displacements because unlike SOC it uses known source locations.
Although DML seems good for fixing the array position, this depends on the array orientation. This arises from a strong correlation of X displacement and angle coefficients $c_0$, $c_1$, $c_2$, $c_3$. Figure 6 plots the DML cost function versus $c_0$ & $\delta_{x0}/L$ and $c_0$ & $\delta_{y0}/L$. Almost equally good cost functions are achieved with moving the array in the X direction and changing the array orientation. This is easily understood geometrically as illustrated in Figure 7. The towed array can be moved along its length and still focus on sources by changing its orientation. Also illustrated is the insensitivity to range reflected in the broad cost function in Figures 5 and 6 with respect to sideways displacement (Y direction).

More frequencies and source locations further apart only slightly decrease X versus orientation correlations. More effective is to use a longer observation time for which there is a greater spread of array positions contributing to the cost function. This is seen in Figure 8 where the cost function is plotted versus $\delta_{x0}/L$ and $c_0$ using 80 seconds of data compared to 10 seconds in Figure 6.

Figures 5, 6 and 8 are for somewhat unrealistic simulated data. Adding uncorrelated white noise to all hydrophones “fills in” the cost function minima and makes the optimal solution parameters less accurate.

Another issue with real data is that a particular frequency source may radiate over an extended area, or there may be more than one source with the same frequency. Consider two sources transmitting at 790 Hz, one with amplitude 100 Pa situated at $X=15$ m,
and the other with amplitude 20 Pa situated at $X = 30$ m. Using data from time interval 135s to 145s, Figure 9 plots the DML cost function versus X and Y array error displacements. (0, 0) is the correct array position. The optimum is biased in the forward X direction when only the weakest source location is known. The best result is obtained when both source locations are known, although only knowing the position of the strongest source also gives the correct array position.

9 TOWED ARRAY SHAPE AND POSITION DETERMINATIONS

Towed array shapes and positions were estimated using both SOC and DML cost functions. Figure 10 compares SOC and DML optimised towed array estimates from 100s to 180s data. Both cost functions improve on the towed array prediction from the hydrodynamic model, although DML is more accurate. Figure 11 shows that DML has the correct 980 Hz source location from the fitted towed array shape, which is not surprising since it is an input for DML, whereas SOC puts the two strong sources about 15 m further aft but still resolves them.

Figure 6. DML cost function versus $c_0$ and $\delta_{x0}/L$ (top) and $c_0$ and $\delta_{y0}/L$ (bottom) for data from 135s to 145s, frequencies 100Hz, 200Hz, 500Hz, and 980Hz. Colour scale is log_{10}(Cost).

Figure 7. Origin of the correlation of towed array position and orientation estimates with similar source focussing performance.

Figure 8. DML cost function versus $c_0$ and $\delta_{x0}/L$ array displacement for data from 100s to 180s, frequencies 100Hz, 200Hz, 500Hz, and 980Hz. Colour scale is log_{10}(Cost).
10 CONCLUSIONS AND DISCUSSION

The main conclusions from this work are as follows:

1. The image contrast data method SOC is not, on its own, sufficient to uniquely determine the position and shape of a towed array during a tow-vessel manoeuvre. There is a range of array shapes and locations that will result in an image contrast very similar to that of the image obtained when the true array shape and location is used.
2. Beamforming using towed array shapes and locations from the SOC method will result in an image that will sharply focus sources, even those at frequencies that weren't used in the inversion, but will in most cases place them at the wrong position along the vessel.
3. If one or more sources are at a known position on the vessel then the DML cost function can be used in place of the contrast. This however does not avoid a strong correlation between the lengthwise position of the array and the array slope.
4. Beamforming with any DML solution will produce a sharply focussed image with all sources in their correct locations.
5. Increasing the duration of the data used in the optimisation reduces the size of the ambiguous region and makes it more likely that the inverted array shape and location at the centre of the data period will be close to the truth.
6. Increasing the number of known sources and/or number of frequencies appears to improve the performance of the DML cost function by removing local minima.
7. Evaluating the DML cost function is computationally much simpler than the preliminary beam forming step required for calculation of the second order contrast, so DML based optimisation is significantly faster than SOC.

For an application in a rapidly changing manoeuvre where the towed array shape and location is needed in real time for far field target detection, methods of reducing the ambiguities are needed. Towed arrays generally have one or more heading sensors that could augment the cost function with an extra term similar to Equation (13), for example, a term based on the mean squared error between the measured and modelled heading sensor outputs.

Increasing the effective data duration for evaluating the cost function, and taking into account the time dependence of parameter vector $\theta$, could be implemented for a real time system by tracking the towing submarine. Tracking was tested with a Kalman filter but performed poorly because of non-Gaussian errors and parameter correlations that violate the underlying assumptions of the filter. Alternative methods such as a fast Gibbs sampler [13] to provide an estimate of the probability distribution of the parameter...
estimates produced by the optimisation algorithm, and a particle filter tracker [14] may work much better.

Finally we mention a couple of limitations of this current work that would need to be overcome before it is possible to apply it to real data. Our analysis is constrained to 2-D where the towed array and submarine are in the same plane. Array cant is generally unavoidable owing to a mismatch of water and array densities, and it changes through a manoeuvre. A full 3-D version of this work, ideally incorporating heading and depth sensor data, would be needed for practical applications.

It should also be mentioned that this work exploits frequencies both below and above the acoustic aperture design frequency. This was possible in the original work by Duncan [5] using the analogue Narama towed array. However current operational digital towed arrays limit their frequency range to the design frequency appropriate to the aperture. Although high frequency octaves may exist, their short apertures cannot achieve the spatial resolution assumed in this paper. A large oversampling rate is needed to achieve with a digital array the spatial resolution of an analogue array. Improving existing digital towed array location and shape estimation with acoustic imaging would probably still require heading and depth sensors, long snapshot sequences and possibly tracking to overcome the disadvantage of no frequencies above an octave design frequency.

11 REFERENCES

[8] T. Rafik, "Compensation for Synthetic Aperture Sonar Motion Errors Using Autofocusing Technique – Simulation Results", School of Product and Engineering