INTERFEROMETRIC IMAGING RADAR FOR SPACE SITUATIONAL AWARENESS
SATELLITE CHARACTERISATION

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ABSTRACT
The three-dimensional imaging of satellites from a ground-based radar is considered in a space situational awareness context. Utilizing the synthetic aperture radar (SAR) tomographic formulation and a model of the collection geometry in a specific low Earth orbit scenario, the shape and repeat nature of collection passes from ascending and descending orbits is evaluated and constraints on the formation of 3D SAR images are discussed. Processing of the three dimensional data can be implemented using interferometric and beamforming techniques. A worked example of a Ku-Band fine resolution radar systems illustrates that both interferometric and the curved collection path depth of focus information can be potentially utilized to form high fidelity three-dimensional images.

Index Terms— Interferometric SAR, Space situational awareness

1. INTRODUCTION
Space situational awareness requires the detection and characterisation of satellites and launch debris in orbit around the Earth. Fine resolution radar imaging using ground-based wide bandwidth radar systems is an established technique for generating range-Doppler images of a satellite [1]. If the satellite is three axis stabilised with a constant attitude with respect to the Earth, such as nadir looking, the Doppler information is derived from the relative rotation of the satellite as it passes over a ground-based system. The two-dimensional image can be analysed to determine satellite attributes including size, shape, attitude, scattering centres and rotation rates. Use of polarimetric radar can enhance the backscatter from small surface discontinuities to “fill in” the satellite image and aid in the classification of specific scattering centres. In addition three-dimensional images can be generated from a series of range-Doppler images collected when the satellite presents different viewing geometries to the radar and processed coherently to a particular look angle [1].

The radar imaging of rotating objects [2] can be formulated using a tomographic approach [3] which provides a spatial domain representation that is an alternative to the time-domain Doppler approach. Section 2 provides an overview of the tomographic approach including its application to three-dimensional radar imaging [4]. This provides a framework for analysis of a ground-based radar illuminating an orbiting satellite. Section 3 develops the data collection geometry for a satellite in low-earth polar orbit that is used to show possible collection of three-dimensional imaging information using physical and repeat pass interferometric approaches. Section 4 describes several approaches for resolving scatterer location and complex reflectivity from the three-dimensional data set. The paper concludes with calculation of parameters for a Ku-Band radar.

2. TOMOGRAPHIC SAR FORMULATION
The formation of fine resolution synthetic aperture radar (SAR) images requires a relative motion between the radar system and object or scene being imaged. This relative motion can be accomplished by the radar flying past a scene, by a static radar illuminating a rotating object or a combination of the two. If the relative motion is viewed from a nominal coordinate system fixed to the object or scene then the radar system traces out a trajectory with a changing elevation and/or azimuth angle.

For fine resolution imaging the changing viewing geometry results in scattering centres moving through both range and cross range resolution elements. Walker [2] showed how dechirped echo data, arranged on a polar grid, represents the Fourier transform of the object reflectivity motion-compensated to the scatterer centre. Munson et. al. [3] drew the linkage between the polar format processing and the polar-slice theorem used in computer-aided tomography. This theorem states that the Fourier transform of a projection at angle \( \theta \) is a “slice” of the two-dimensional Fourier transform of properties of the object being measured, also at angle \( \theta \). This was further extended into three dimensions by Jakowatz & Thompson [4].
Radar observations of an object measures the reflectivity at range bins which are the sum of all backscatter from the object at that range. Consequently the range echo is a projection of object reflectivity at elevation $\psi$ and azimuth $\theta$ onto the line-of-sight (LOS) vector.

In the case of imaging radar the projection is formed by the fine resolution range bins where, for a single pulse, all the backscatter from the scene or object in a range bin is summed into a single measurement. Fig. 1 shows the radar line-of-sight (LOS) to a three-dimensional object located in a local coordinate system. The range echo has measurements at each range bin corresponding to the summation of the backscatter from the object, $g(x, y, z)$, at a specific range. This summation process is the projection of the object reflectivity onto the radar LOS vector. If the radar is at sufficient range compared to the object size then the constant range surface can be adequately approximated by a plane (rather than a spherical wave front). The Fourier transform of the radar echo samples collected at $(\psi, \theta)$ are samples of the three-dimensional Fourier transform $G(k_x, k_y, k_z)$ of the backscatter $g(x, y, z)$ evaluated at a line in Fourier space defined by the polar angle $(\psi, \theta)$ [4].

The radar is measuring the backscatter properties of the object at some bandwidth B around the radar centre frequency, $\omega/\lambda$, where $\omega$ is the speed of light and $\lambda$ is the radar wavelength. The Fourier transform of the radar echo in the radar LOS is located around the centre frequency and so is offset from the wavenumber $k$-space (Frequency domain) origin. As the radar moves past the object $\psi$ and $\theta$ vary and the collected lines map out a surface in the three-dimensional $k$-space. Multiple interferometric observations of the object from geometries slightly offset orthogonal to the radar motion results in a stack of surfaces in the $k$-space domain as shown in Fig. 2 [5]. These surfaces can be collected through repeat pass collections or from a single pass if the radar system has multiple receive antennas to yield $n=1,2,\ldots,N$ surfaces at nominal elevation angle $\psi_n$. Each range echo is centered at $4\pi\lambda$ with linear extent $4\pi B/c$ and located at polar angle $(\psi_n, \theta)$. The angular width subtended by a single radar collection is $\Delta \phi$ (this is smaller than the subtended polar angle $\Delta \theta$) and from multiple collections at the same azimuth is $\Delta \psi = \psi_n - \psi_1$ in the orthogonal height direction. The edges of the collection volume define a spatial-frequency support $W(k_x, k_y, k_z)$ of the object’s $k$-space data $G(k_x, k_y, k_z)$. This data support is typically a wedge in the spatial-frequency domain that is non-rectangular and non-uniformly sampled. The corresponding spatial resolution is $2\pi/\Delta k$ where $\Delta k$ is the corresponding Fourier domain bandwidth extent. The extents have approximate spatial bandwidths and resolutions in the range, azimuth and height directions of.

| Range      | $\Delta k_r \approx 4\pi B/c$, $\delta_r \approx c/2B$ |
| Azimuth    | $\Delta k_a \approx 4\pi \Delta \phi \lambda$, $\delta_a \approx \lambda/2\Delta \phi$ (1) |
| Height     | $\Delta k_h \approx 4\pi \Delta \psi \lambda$, $\delta_h \approx \lambda/2\Delta \psi$ |

The actual achieved resolutions are slightly less than these values when a rectangular volume is chopped out of the collected wedge shaped volume. The outer boundary of the collected volume defines the resolution and the number of samples across each dimension of the volume defines the spatial extent of the image. If there are N collections then the spatial extent is $(N-1)\delta$. The most difficult dimension to achieve significant number of collections is the height dimension.

Figure 2: Frequency domain representation of the collected data. A single echo is a line in the Frequency domain (range extent $4\pi B/c$), a series of echoes forms a collection surface (slant plane subtended angle $\Delta \phi$) and a series of collections at different elevation angle forms a collection volume (height $\Delta \psi$). The collection volume is the three-dimensional aperture of support, $W(k_x, k_y, k_z)$. 

Figure 1: Radar observations of an object measures the reflectivity at range bins which are the sum of all backscatter from the object at that range. Consequently the range echo is a projection of object reflectivity at elevation $\psi$ and azimuth $\theta$ onto the line-of-sight (LOS) vector.
3. SATELLITE IMAGING GEOMETRY

As a satellite passes overhead on its orbit a ground-based radar system will view the satellite from a variety of directions. The imaging geometry is defined by the radar LOS as it traces out a path in the coordinate system attached to the satellite (Fig 1) to generate the frequency domain support for the radar image (Fig 2). There is a wide range of satellite types (e.g. three axis stabilized and spinning), orbit geometries and locations for a ground-based radar. Here a specific simple scenario is developed that illustrates the major attributes of the satellite imaging geometry.

The scenario shown in Fig 3 has several simplifying features. The satellite is in a 90° circular polar orbit, so that the satellite crosses over the Earth’s north and south poles. The radar is located at the equator (intersection with the x axis in the diagram) so it views the satellite traveling from north to south and south to north on descending and ascending parts of the orbit respectively. The orbital plane is assumed fixed in inertial space so the radar, that is moving on the spinning Earth (angular velocity \( \omega_e \)), has a velocity component to the east. This is represented in Fig. 3 with the angle between the orbital plane and x axis being a function of time, \( \beta(t) \). The satellite is three axis stabilized with a local coordinate frame having \( z_s \) in the nadir direction to the centre of the Earth, \( x_s \) in the satellite orbital velocity direction and \( y_s \) in an orthogonal cross track direction. The imaging geometry, defined by the radar LOS trace in the local satellite coordinate system, can be described by the elevation and azimuth, \( \psi \) and \( \theta \), shown in Fig 1. These angles are calculated from a numerical simulation of the time history of \( r(t) \) in the spacecraft local coordinate frame \((x_s, y_s, z_s)\).

3.1. Repeat Pass Interferometric Collections

The specific scenario modeled here is a low Earth orbit satellite at 800 km altitude above a spherical Earth in an orbit with \( \beta = 5° \) and 175° when the satellite crosses the equator on the ascending and descending passes respectively (to view the same side of the satellite). The radar LOS trace for ascending and descending passes is shown in Fig. 4 for approximately 10 minutes around the equator crossing. Due to the rotation of the radar system under the orbital plane the maximum elevation is offset from the equator crossing time and so is displaced in different azimuth directions from the \( y_s \) axis for the ascending and descending passes with the cross over point occurring when the satellite crosses the equator (azimuth = 0°).
(depends on the choice of radar frequency), data collection over a wide range of azimuth angles can be processed into interferometric pairs with different Δψ baselines. The curved collection surface shown in Fig. 4 also provides an out of collection surface depth of focus parameter [6] that may provide a further height discriminate depending on the choice of radar frequency and projected height of the spacecraft. For objects located Δh out of the imaging plane the impulse response function mainlobe width broadens by a factor of approximately 16 Δψ, Δh / λc, where Δψc is here the change in elevation angle over the collection surface [6].

Additional repeat pass collections can be obtained at slightly different β values to collect data over the k-space volume shown in Fig. 2. Depending on the satellite orbit these may occur at significant time spacing (e.g. days or months) and may even be infeasible if the satellite orbit is tightly controlled. Fig. 5 shows the collection surface envelop of two ascending and two descending orbits with β = 5º and 4.4º for the descending passes to yield a Δψ ~ 3º between collection surfaces. Further collections obtained between these limits will fill in the frequency space collection volume with the ideal circumstance being collections spaced at regular Δψ increments.

3.2. Single Pass Interferometric Collections

An alternative to repeat pass collections is to have an array of receive dishes at the radar that would collect the multiple surfaces from a single pass. This antenna array could have one transmit/receive radar and N-1 receive only radars to collect N surfaces. The location of the radar echo line in the frequency domain is at the same angle as the bisector of the transmit and receive LOS vectors [6]. To achieve the same Δψ ~ 3º in Fig. 5 requires a receiver with a LOS vector to the satellite approximately 6º from the transmitter. In the scenario used here the difference in β can be achieved through N antennas distributed along the equator. A Δβ ~ 1.2º is required to achieve a Δψ ~ 3º when the transmitter is located at β=5º. This corresponds to an array of antennas distributed over approximately 130 km. The minimum array spacing for a Δψ ~ 3º occurs for a directly overhead pass (transmitter β=0º) and is ~84 km.

4. ANALYSIS OF THE THREE-DIMENSIONAL DATA

The acquired collection surfaces in Fig. 2 can be modeled and processed according to the traditional tomographic formulation for complex SAR data [3] with extension to three dimensions [5,7] and proper accounting for the bistatic angle for a single pass system. Fourier-inverting this model of the collected data, the scene reflectivity can be estimated as

\[ \hat{g}(x, y, z) = \iiint G(k_x, k_y, k_z) e^{i(k_x x + k_y y + k_z z)} W(k_x, k_y, k_z) dk_x dk_y dk_z, \]  

(2)

where the 3D aperture of support \( W(k_x, k_y, k_z) \) allows for dependencies between the spatial-frequency dimensions \( k_x, k_y \), and \( k_z \). In particular \( W \) defines the shape and extent of the usable collected data, and so determines the properties of the point-spread function, including the resolution and the aliasing extents in each direction.

Figure 5: Radar LOS trace in spacecraft coordinate frame elevation and azimuth for β=5º (β=175º) and β=4.4º (175.6º) at the equator crossing for the descending (ascending) pass, and 800 km orbit altitude. The time extent is 30 seconds and the x axis is the slant plane SAR angle φ in the collection surface.

The extent of the wedge in the height direction, orthogonal to both range and azimuth, can be viewed as a synthetic aperture in height or alternatively as \( N \) array elements in an elevation beamforming array. In [5] the support aperture \( W \) is re-written in terms of azimuth spatial-frequency, slant range spatial-frequency and elevation angle ψ. Consequently the wedge may be processed by forming \( N \) slant-plane images using efficient FFT-based algorithms such as the polar-format algorithm, then coherently combining the image stack to resolve the scattering position in height. For each pixel location \( (x, y, z = z_p) \) in the output image at a chosen focus height \( z_p \) the corresponding pixel location in each of the slant-plane images that constitute the stack must be determined, and then a pixel value calculated by some method of interpolation. These calculated pixels are then coherently combined by applying a phase factor and summing. For the \( n^{th} \) pixel \( n = 1, 2, \ldots, N \), the phase factor is
\[ \Phi_{\text{synth}}(y, z, n) = k_0(y + z \tan \psi_a) \cos \psi_a, \]
\[ k_0 = 4\pi / \lambda, \]

where \( k_0 \) is the \( k \)-space offset to the centre of each collection aperture. In this way, azimuth-range images focused to a variety of heights can be produced, and these images stacked to give height-range or height-azimuth representations of the scene.

The effective use of a vector of phase weights to combine array elements may be interpreted as a beamforming approach to the problem. Indeed, more advanced beamforming solutions can be applied, where effectively each output pixel at a particular focus height is the output from a steered beam formed using an input vector of corresponding pixels from the constituent SAR images and an appropriate weight vector. Of particular interest is the minimum-variance distortionless response (MVDR) beamformer (a.k.a. Capon beamformer), which under particular conditions is optimal, as it adaptively minimizes the output power from all directions (target heights) except the steered direction (focus height) \([8]\). Up to \( N - 1 \) nulls are steered onto strong returns from off-focus-height scatterers, potentially overcoming the poor point-spread function that results from Fourier-based processing of the sparse, irregular data in \( k_z \) and greatly improving the effective resolution \([9]\). The trade-off is that such an optimal technique is much more sensitive to noise fluctuations and system errors, or rather to deviations from the data model.

The vector of weights \( \mathbf{w}_{\text{MVDR}} \) is calculated as

\[ \mathbf{w}_{\text{MVDR}}(x, y, z_f) = \frac{R^{-1}v}{\sqrt{v^H R^{-1}v}}, \]

where \( R \) is the covariance matrix of the \( N \) input pixels. A second draw-back of such an adaptive approach is now apparent, in that the reliable calculation of \( R^{-1} \) for every output pixel is non-trivial \([10]\).

### 5. POSSIBLE IMPLEMENTATIONS

In Australia there is a 2 GHz frequency allocation in the Ku-Band from 15.7 GHz to 17.7 GHz for radio location. Using this full bandwidth would achieve a range resolution of approximately 10 cm after application of a Hamming window to control sidelobes from closely spaced, potentially bright radar scattering centres (<-40dB first sidelobe). Achieving a comparable resolution in the azimuth and height dimensions requires approximately 6.7° of synthetic aperture (\( \Delta \theta \) and \( \Delta \psi \)). At the satellite equator crossing point with an orbit offset of \( \beta = 5^\circ \) the time to generate this synthetic aperture is approximately 17 seconds. If the unambiguous cross range extent of the scene is 100 m then at 0.1 m resolution approximately 1000 measurements are required, resulting in a minimum radar pulse rate of ~60 Hz. Achieving a similar resolution and reasonable unambiguous extent, say 20m, in the height dimension is difficult to implement (200 repeat passes or array elements). One alternative is to take advantage of the curved collection surface to assist with removing ambiguities in the height dimension. The collection surface in Fig. 5 has an out of plane elevation deviation \( \Delta \psi_c = 0.14^\circ \). The impulse response mainlobe broadening factor is ~2.2 per metre of out of plane height. Employing interferometric and depth of focus methods provides a potential mechanism to reduce the number of collection surfaces required for high fidelity three-dimensional imaging.

### 6. SUMMARY

The three-dimensional imaging of satellites from a ground-based radar is addressed using the synthetic aperture radar tomographic formulation. A model of the collection geometry, implemented for a specific scenario of a low Earth orbit satellite, shows the shape and repeat nature of collection passes from ascending and descending orbits. The processing of the three-dimensional data is considered using interferometric and beamforming techniques. A worked example of a Ku-Band fine resolution radar system illustrates that both interferometric and the curved collection path depth of focus information can be potentially utilized to form a high fidelity three-dimensional image.

### 11. REFERENCES
